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Fixed point results of generalized $\alpha - (\psi, \phi) - Z_G-$ contractive mappings on non-Archimedean modular metric spaces

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Abstract: In this paper, by using the concept of α-admissible mapping and generalized class of simulation functions, we establish the existence and uniqueness of fixed point of a generalized $\alpha - (\psi, \phi) - Z_G-$ contractive mappings on non-Archimedean modular metric spaces. Our results generalize and extend various comparable results in the existing literature.

Keywords: Non-Archimedean modular metric space, Simulation functions, $\alpha-$admissible mapping, Weak contraction.

MSC: 47H10, 54H25.

1 Introduction

Let (X, d) be a metric space. A mapping $f : X \longrightarrow X$ is a contraction if there exists a constant $k \in (0, 1)$ such that

$$d(fx, fy) \leq kd(x, y)$$

holds for any $x, y \in X$. If X is complete, then every contraction has a unique fixed point and that point can be obtained as a limit of repeated iteration of the mapping at any point of X (the Banach contraction principle). Obviously, every contraction is a continuous function.

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A mapping $f : X \longrightarrow X$ is a $\phi-$ weak contraction if for each $x, y \in X$, there exists a function $\phi : [0,\infty) \longrightarrow [0,\infty)$ such that ϕ is positive on $(0,\infty)$, $\phi(0) = 0$, and

$$d(fx, fy) \leq d(x, y) - \phi(d(x, y)).$$

The concept of the weak contraction was defined by Alber and Guerre-Delabriere [2] in 1997. They defined such mappings for single-valued maps on Hilbert spaces and proved the existence of fixed points. Rhoades [20] showed that most results of [2] are true for any Banach space. Also Rhoades proved the following generalization of the Banach contraction principle.

Theorem 1.1. *Let (X, d) be a complete metric space and $f : X \longrightarrow X$ be a $\phi-$weak contraction on X. If ϕ is a continuous and nondecreasing function with $\phi(t) > 0$ for all $t > 0$ and $\phi(0) = 0$, then f has a unique fixed point.*

Every contraction is a $\phi-$weak contraction if we take $\phi(t) = kt$,where $0 < k < 1$.
Dutta and Choudhury [9] proved the following generalization of Theorem 1.1.

Theorem 1.2. *Let (X, d) be a complete metric space and $f : X \longrightarrow X$ be a self-mapping satisfying the inequality*

$$\psi(d(fx, fy)) \leq \psi(d(x, y)) - \phi(d(x, y)),$$

where $\psi, \phi : [0,\infty) \longrightarrow [0,\infty)$ are both continuous and nondecreasing functions with $\psi(t) = \phi(t) = 0$ if and only if $t = 0$.Then f has a unique fixed point.

Doric [8] generalized Theorem 1.2 as follows.

Theorem 1.3. *Let (X, d) be a complete metric space and $f : X \longrightarrow X$ be a self-mapping satisfying the inequality*

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)),$$

where

$$M(x, y) = \max\left\{d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2}\right\}$$

and

a. $\psi : [0,\infty) \longrightarrow [0,\infty)$ is a continuous monotone nondecreasing function with $\psi(t) = 0$ if and only if $t = 0$,

b. $\phi : [0,\infty) \longrightarrow [0,\infty)$ is a lower semi-continuous monotone function with $\phi(t) = 0$ if and only if $t = 0$.

Then f has a unique fixed point.

Further, many authors improved and generalized $Z-$contraction in abstract spaces [11, 18, 26, 16, 4, 13, 27, 21, 22, 23, 14].

Let X be a nonempty set and $\omega : (0,\infty) \times X \times X \longrightarrow [0,\infty]$ be a function; for simplicity, we will write:

$$\omega_\lambda(x, y) = \omega(\lambda, x, y)$$

for all $\lambda > 0$ and $x, y \in X$.

Definition 1.4. *A function $\omega : (0,\infty) \times X \times X \longrightarrow [0,\infty]$ is called a modular metric on X if the following axioms hold:*

i. $x = y$ *if and only if* $\omega_\lambda(x,y) = 0$ *for all* $\lambda > 0$;

ii. $\omega_\lambda(x,y) = \omega_\lambda(y,x)$ *for all* $\lambda > 0$ *and* $x, y \in X$;

iii. $\omega_{\lambda+\mu}(x,y) \leq \omega_\lambda(x,z) + \omega_\mu(z,y)$ *for all* $\lambda,\ \mu > 0$ *and* $x, y, z \in X$.

In the above definition, if we utilize the condition:

i_1. $\omega_\lambda(x,x) = 0$ for all $\lambda > 0$ and $x \in X$

instead of (i), then ω is said to be a pseudomodular metric on X. A modular metric ω on X is called regular if the following weaker version of (i) is satisfied:

$x = y$ if and only if $\omega_\lambda(x,y) = 0$ for some $\lambda > 0$.

Again, ω is called convex if for $\lambda,\ \mu > 0$ and $x, y, z \in X$, the inequality holds:

$$\omega_{\lambda+\mu}(x,y) \leq \frac{\lambda}{\lambda+\mu}\omega_\lambda(x,z) + \frac{\mu}{\lambda+\mu}\omega_\mu(z,y).$$

Definition 1.5. *[5, 6] suppose that* ω *be a pseudomodular on* X *and* $x_0 \in X$ *and fixed. Therefore, the two sets:*

$$X_\omega = X_\omega(x_0) = \{x \in X : \omega_\lambda(x,x_0) \quad as \quad \lambda \longrightarrow \infty\}$$

and

$$X^*_\omega = X^*_\omega(x_0) = \{x \in X : \exists\lambda = \lambda(x) > 0 \quad such\ that\ \omega_\lambda(x,x_0) < \infty\}.$$

X_ω *and* X^*_ω *are called modular spaces (around* x_0*).*

It is clear that $X_\omega \subset X^*_\omega$, but this inclusion may be proper in general. Suppose that ω is a modular on X; from [5, 6], we derive that the modular space X_ω can be equipped with a (nontrivial) metric, induced by ω and given by:

$$d_\omega(x,y) = \inf\{\lambda > 0 : \omega_\lambda(x,y) < \lambda\}$$

for all $x, y \in X_\omega$.

Note that if ω is a convex modular on X, then according to [5, 6], the two modular space coincide, i.e., $X_\omega = X^*_\omega$, and this common set can be endowed with the metric d^*_ω given by:

$$d^*_\omega(x,y) = \inf\{\lambda > 0 : \omega_\lambda(x,y) < 1\}$$

for all $x, y \in X_\omega$. Such distances are called Luxemburg distances.

Definition 1.6. *[15] Let* X_ω *be a modular metric space,* M *a subset of* X_ω *and* $(x_n)_{n\in\mathbb{N}}$ *be a sequence in* X_ω. *Therefore:*

1. $(x_n)_{n\in\mathbb{N}}$ *is called* $\omega-$*convergent to* $x \in X_\omega$ *if and only if* $\omega_\lambda(x_n,x) \longrightarrow 0$ *as* $n \longrightarrow \infty$ *for all* $\lambda > 0$. x *will be called the* $\omega-$*limit of* (x_n).

2. $(x_n)_{n\in\mathbb{N}}$ *is called* $\omega-$*Cauchy if* $\omega_\lambda(x_n,x_m) \longrightarrow 0$, *as* $m, n \longrightarrow \infty$ *for all* $\lambda > 0$.

3. M *is called* $\omega-$*closed if the* $\omega-$*limit of* $\omega-$*convergent sequence of* M *always belong to* M.

4. *M is called $\omega-$complete if any $\omega-$Cauchy sequence in M is $\omega-$convergent to a point of M.*

5. *M is called $\omega-$bounded if for all $\lambda > 0$, we have*

$$\delta_\omega(M) = \sup\{\omega_\lambda(x,y); x,y \in M\} < \infty.$$

Recently, Paknazar et al. [17] introduced the following concept.

Definition 1.7. *In Definition 1.4, if we replace (iii) by:*

iv. $\omega_{\max\{\lambda,\mu\}}(x,y) \leq \omega_\lambda(x,z) + \omega_\mu(z,y)$

for all $\lambda,\ \mu > 0$ and $x,y,z \in X_\omega$.

Then, X_ω is called the non-Archimedean modular metric space. Since (iv) implies (iii), every non-Archimedean modular metric space is a modular metric space.

Definition 1.8. *[13] A simulation function is a mapping $\zeta : [0,\infty)^2 \longrightarrow \mathbb{R}$ satisfying the following conditions:*

ζ_1. $\zeta(0,0) = 0$,

ζ_2. $\zeta(s,t) < s - t$, *for all* $t,s > 0$,

ζ_3. *if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim\limits_{n\longrightarrow\infty} t_n = \lim\limits_{n\longrightarrow\infty} s_n = l$, $l \in (0,\infty)$ then* $\limsup\limits_{n\longrightarrow\infty} \zeta(t_n,s_n) < 0$.

Argoubi et al.[4] shown that the condition (ζ_1) to be redundant one in the above definition of simulation function and so redefined it as:

Definition 1.9. *A simulation function is a mapping $\zeta : [0,\infty)^2 \longrightarrow \mathbb{R}$ satisfying the following:*

i. $\zeta(s,t) < s - t$, *for all* $t,s > 0$,

ii. if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0,\infty)$ such that $\lim\limits_{n\longrightarrow\infty} t_n = \lim\limits_{n\longrightarrow\infty} s_n > 0$, and $t_n < s_n$, then $\limsup\limits_{n\longrightarrow\infty} \zeta(t_n,s_n) < 0$.

Example 1.10. *[4] Let $\zeta_p : [0,\infty) \times [0,\infty) \longrightarrow \mathbb{R}$ be function defined by*

$$\zeta_p(t,s) = \begin{cases} 1 & \text{if } (t,s) = (0,0), \\ ps - t & \text{otherwise,} \end{cases}$$

where $p \in (0,1)$. Then ζ_p is a simulation function.

For examples and related results on simulation functions, one may refer to [11, 18, 26, 16, 4, 13, 27, 19, 24, 1].

Definition 1.11. *[3] A mapping $G : [0,\infty)^2 \longrightarrow \mathbb{R}$ is called $C-$class function if it is continuous and satisfies the following conditions:*

i. $G(s,t) \leq s$,

ii. $G(s,t) = s$ *implies that either* $s = 0$ *or* $t = 0$*, for all* $s,t \in [0,\infty)$.

Definition 1.12. *[18] A* C_G-*simulation function is a mapping* $\zeta : [0,\infty)^2 \longrightarrow \mathbb{R}$ *satisfying the following:*

a. $\zeta(t,s) < G(s,t)$ *for all* $t,s > 0$*, where* $G : [0,\infty)^2 \longrightarrow \mathbb{R}$ *is a* $C-$*class function,*

b. if $\{t_n\}$ *and* $\{s_n\}$ *are sequences in* $(0,\infty)$ *such that* $\lim_{n\longrightarrow\infty} t_n = \lim_{n\longrightarrow\infty} s_n > 0$*, and* $t_n < s_n$*, then* $\limsup_{n\longrightarrow\infty} \zeta(t_n,s_n) < C_G$.

Definition 1.13. *[18] A mapping* $G : [0,\infty)^2 \longrightarrow \mathbb{R}$ *has a property* C_G*, if there exists a* $C_G \geq 0$ *such that*

i. $G(s,t) > C_G$ *implies* $s > t$*,*

ii. $G(t,t) \leq C_G$ *for all* $t \in [0,\infty)$.

Let Z_G denotes the family of all C_G-simulation functions $\zeta : [0,\infty)^2 \longrightarrow \mathbb{R}$.

Definition 1.14. *[18] Let* (X,d) *be a metric space and* $f,g : X \longrightarrow X$ *be self-mappings. The mapping* f *is called a* $(Z_G,g)-$*contraction if there exists* $\zeta \in Z_G$ *such that*

$$\zeta(d(fx,fy), d(gx,gy)) \geq C_G$$

for all $x,y \in X$ *with* $gx \neq gy$.

If $g = i_X$(identity mapping on X)and $C_G = 0$, we get $Z-$contraction of [13].

Definition 1.15. *[18] Let* (X,d) *be a metric space and* $f,g : X \longrightarrow X$ *be self-mappings. The mapping* f *is called a generalized* $(Z_G,g)-$*contraction if there exists* $\zeta \in Z_G$ *such that*

$$\zeta\left(d(fx,fy), \max\left\{d(gx,gy), d(gx,fx), d(gy,fy), \frac{d(gx,fy)+d(gy,fx)}{2}\right\}\right) \geq C_G,$$

for all $x,y \in X$ *with* $gx \neq gy$.

If $g = i_X$(identity mapping on X)and $C_G = 0$, we get $Z-$contraction of [16].

Recently, Samet et al. [25] introduced the concept of $\alpha-$admissible mappings and prove some fixed point results.

Definition 1.16. *[25] For a nonempty set* X*, let* $f : X \longrightarrow X$ *and* $\alpha : X \times X \longrightarrow [0,\infty)$ *be mappings. We say that the self-mapping* f *on* X *is* $\alpha-$*admissible if for all* $x,y \in X$*, we have*

$$\alpha(x,y) \geq 1 \quad \Rightarrow \quad \alpha(fx,fy) \geq 1. \tag{1.1}$$

Definition 1.17. *[25]Let* (X,d) *be complete metric spaces and* $f : X \longrightarrow X$ *be an given mapping. We say that* f *is an* $\alpha - \psi-$*contractive mapping if there exists* $\alpha : X \times X \longrightarrow [0,\infty)$ *such that*

$$\alpha(x,y)\, d(fx,fy) \leq \psi(d(x,y))$$

for all $x,y \in X$.

Theorem 1.18. *[25] Let (X, d) be a complete metric space and $f : X \longrightarrow X$ be an $\alpha - \psi-$contractive mapping satisfying the following conditions:*

i. f is $\alpha-$admissible,

ii. there exists $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$,

iii. f is continuous.

Then, f has a fixed point.

Example 1.19. *[12] Let $X = [0, 1]$ and define $f : X \to X$ and $\alpha : X \times X \longrightarrow [0, \infty)$ by*

$$fx = \begin{cases} \frac{1}{4}, & x \in [0,1), \\ 0, & x = 1 \end{cases}$$

and

$$\alpha(x,y) = \begin{cases} 1, & (x,y) \in \left(\left[0,\frac{1}{4}\right] \times \left[\frac{1}{4},1\right]\right) \cup \left(\left[\frac{1}{4},1\right] \times \left[0,\frac{1}{4}\right]\right), \\ 0, & \text{otherwise.} \end{cases}$$

Then, f is a $\alpha-$admissible.

Definition 1.20. *[11] Let f be a self-mapping on a metric space X endowed with metric d. Let $\alpha : X \times X \longrightarrow [0, \infty)$ be such that*

$$\zeta(\alpha(x,y) d(fx, fy), d(x,y)) \geq 0,$$

for all $x, y \in X$, then f is called $\alpha-$admissible $Z-$contraction with respect to ζ with $\zeta \in Z$.

Also Karapinar [11] investigated the existence and uniqueness of fixed points of certain mappings via simulation functions in the context of complete metric spaces.

2 Main Results

In the sequel the function ω is convex and regular. We denote by Ψ the set of functions $\psi : [0, \infty) \longrightarrow [0, \infty)$ satisfying the following hypotheses:

ψ_1. ψ is continuous and nondecreasing,

ψ_2. $\psi(t) = 0$ if and only if $t = 0$.

and denote by Φ the set of functions $\phi : [0, \infty) \longrightarrow [0, \infty)$ satisfying the following hypotheses:

ϕ_1. ϕ is lower semi-continuous,

ϕ_2. $\phi(t) = 0$ if and only if $t = 0$.

Definition 2.1. *Let X_ω be a non-Archimedean modular metric space and $f : X_\omega \longrightarrow X_\omega$ and $\alpha : X_\omega \times X_\omega \longrightarrow [0, \infty)$ be mappings. We say that f is generalized $\alpha - (\psi, \phi) - Z_G-$contractive mapping if there exists $\zeta \in Z_G$ such that*

$$\zeta(\psi(\alpha(x,y)\omega_\lambda(fx, fy)), \psi(M(x,y)) - \phi(M(x,y))) \geq C_G, \tag{2.1}$$

where $M(x,y) = \left\{\omega_\lambda(x,y), \omega_\lambda(x,fx), \omega_\lambda(y,fy), \frac{\omega_\lambda(x,fy)+\omega_\lambda(y,fx)}{2}\right\}$, $\psi \in \Psi$, $\phi \in \Phi$, for all $x, y \in X_\omega$.

Theorem 2.2. *Let X_ω be a complete non-Archimedean modular metric space and f be a generalized $\alpha-(\psi,\phi)-Z_G-$contractive mapping. Suppose that the following conditions hold:*

i. f is $\alpha-$admissible mapping,

ii. there exists $x_0 \in X_\omega$ such that $\alpha(x_0, fx_0) \geq 1$,

iii. f is $\omega-$continuous.

Then, f has a fixed point.

Proof. Let $x_0 \in X_\omega$ be such that $\alpha(x_0, fx_0) \geq 1$. Define an iterative sequence $\{x_n\}$ in X by letting $x_{n+1} = fx_n$ for all $n \in \mathbb{N}$. If there exists an n_0 such that $x_{n_0} = x_{n_0+1}$, then $u = x_{n_0}$ becomes a fixed point of f. Consequently, we shall assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. So we get $\omega_\lambda(x_n, x_{n+1}) > 0$, for all $n \in \mathbb{N}$. Regarding that f is $\alpha-$admissible, we derive

$$\alpha(x_0, x_1) = \alpha(x_0, fx_0) \geq 1 \quad \Rightarrow \quad \alpha(fx_0, fx_1) = \alpha(x_1, x_2) \geq 1.$$

Recursively, we obtain that

$$\alpha(x_n, x_{n+1}) \geq 1, \tag{2.2}$$

for all $n \in \mathbb{N}$. From (2.1) and (2.2),

$$\begin{aligned}
&C_G \\
&\leq \zeta\left(\psi\left(\alpha(x_n, x_{n-1})\,\omega_\lambda(fx_n, fx_{n-1})\right), \psi(M(x_n, x_{n-1})) - \phi(M(x_n, x_{n-1}))\right) \\
&= \zeta\left(\psi\left(\alpha(x_n, x_{n-1})\,\omega_\lambda(x_{n+1}, x_n)\right), \psi(M(x_n, x_{n-1})) - \phi(M(x_n, x_{n-1}))\right) \\
&< G\left(\psi(M(x_n, x_{n-1})) - \phi(M(x_n, x_{n-1})), \psi\left(\alpha(x_n, x_{n-1})\,\omega_\lambda(x_{n+1}, x_n)\right)\right).
\end{aligned}$$

Further, using (i) of Definition 1.12, we have

$$\psi\left(\alpha(x_n, x_{n-1})\,\omega_\lambda(x_{n+1}, x_n)\right) < \psi(M(x_n, x_{n-1})) - \phi(M(x_n, x_{n-1})). \tag{2.3}$$

Consequently, we get that

$$\begin{aligned}
\psi(\omega_\lambda(x_{n+1}, x_n)) &\leq \psi\left(\alpha(x_n, x_{n-1})\,\omega_\lambda(x_{n+1}, x_n)\right) \\
&< \psi(M(x_n, x_{n-1})) - \phi(M(x_n, x_{n-1})) \\
&< \psi(M(x_n, x_{n-1})),
\end{aligned} \tag{2.4}$$

where

$$\begin{aligned} M\left(x_n, x_{n-1}\right) &= \max\left\{\omega_\lambda\left(x_n, x_{n-1}\right), \omega_\lambda\left(x_n, fx_n\right), \omega_\lambda\left(x_{n-1}, fx_{n-1}\right), \right. \\ &\left. \frac{\omega_\lambda(x_n, fx_{n-1}) + \omega_\lambda(x_{n-1}, fx_n)}{2}\right\} \\ &= \max\left\{\omega_\lambda\left(x_n, x_{n-1}\right), \omega_\lambda\left(x_n, x_{n+1}\right), \omega_\lambda\left(x_{n-1}, x_n\right), \right. \\ &\left. \frac{\omega_\lambda(x_n, x_n) + \omega_\lambda(x_{n-1}, x_{n+1})}{2}\right\} \\ &= \max\left\{\omega_\lambda\left(x_n, x_{n-1}\right), \omega_\lambda\left(x_n, x_{n+1}\right), \omega_\lambda\left(x_{n-1}, x_n\right), \right. \\ &\left. \frac{\omega_\lambda(x_n, x_n) + \omega_{\max\{\lambda,\lambda\}}(x_{n-1}, x_{n+1})}{2}\right\} \\ &\leq \max\left\{\omega_\lambda\left(x_n, x_{n-1}\right), \omega_\lambda\left(x_n, x_{n+1}\right), \frac{\omega_\lambda(x_n, x_{n-1}) + \omega_\lambda(x_n, x_{n+1})}{2}\right\} \\ &= \max\left\{\omega_\lambda\left(x_n, x_{n-1}\right), \omega_\lambda\left(x_n, x_{n+1}\right)\right\}. \end{aligned}$$

If $M\left(x_n, x_{n-1}\right) = \max\left\{\omega_\lambda\left(x_n, x_{n-1}\right), \omega_\lambda\left(x_{n+1}, x_n\right)\right\} = \omega_\lambda\left(x_{n+1}, x_n\right)$ for some $n \in \mathbb{N}$, then from (2.4), we have a contradiction. So, $M\left(x_n, x_{n-1}\right) = \omega_\lambda\left(x_n, x_{n-1}\right)$ for all $n \in \mathbb{N}$. Consequently, from (2.4) and since ψ is nondecreasing, we gives

$$\omega_\lambda\left(x_{n+1}, x_n\right) < \omega_\lambda\left(x_n, x_{n-1}\right), \tag{2.5}$$

for all $n \in \mathbb{N}$. So $\left\{\omega_\lambda\left(x_{n+1}, x_n\right)\right\}$ is a monotonically decreasing sequence of nonnegative real numbers, and hence there exists $l \geq 0$ such that

$$\lim_{n\longrightarrow\infty} \omega_\lambda\left(x_{n+1}, x_n\right) = l. \tag{2.6}$$

Assume that $l > 0$. Then using (2.1) and (b) of the Definition 1.11, we have

$$\begin{aligned} C_G &\leq \limsup_{n\longrightarrow\infty} \zeta\left(\psi\left(\alpha\left(x_n, x_{n-1}\right)\omega_\lambda\left(fx_n, fx_{n-1}\right)\right), \right. \\ &\left. \psi\left(M\left(x_n, x_{n-1}\right)\right) - \phi\left(M\left(x_n, x_{n-1}\right)\right)\right) \\ &\leq \limsup_{n\longrightarrow\infty} \zeta\left(\psi\left(\alpha\left(x_n, x_{n-1}\right)\omega_\lambda\left(x_{n+1}, x_n\right)\right), \right. \\ &\left. \psi\left(\omega_\lambda\left(x_n, x_{n-1}\right)\right) - \phi\left(\omega_\lambda\left(x_n, x_{n-1}\right)\right)\right) \\ &\leq \limsup_{n\longrightarrow\infty} \zeta\left(\psi\left(l\right), \psi\left(l\right)\right) \\ &< C_G, \end{aligned}$$

which is a contradiction and so $l = 0$.

Now we show that $\{x_n\}$ is a $\omega-$Cauchy sequence. Assume on the contrary. Then there exists $\varepsilon > 0$ such that we can find two subsequences $\{m_k\}$ and $\{n_k\}$ of positive integers satisfying $n_k > m_k \geq k$ such the following inequalities hold:

$$\omega_\lambda\left(x_{n_k}, x_{m_k}\right) \geq \varepsilon, \text{ and } \omega_\lambda\left(x_{n_k}, x_{m_k-1}\right) < \varepsilon. \tag{2.7}$$

From (2.7) and (iv) of Definition 1.7, it follows that

$$\begin{aligned} \varepsilon \leq \omega_\lambda\left(x_{n_k}, x_{m_k}\right) &= \omega_{\max\{\lambda,\lambda\}}\left(x_{n_k}, x_{m_k}\right) \\ &\leq \omega_\lambda\left(x_{n_k}, x_{m_k-1}\right) + \omega_\lambda\left(x_{m_k-1}, x_{m_k}\right) \\ &< \varepsilon + \omega_\lambda\left(x_{m_k-1}, x_{m_k}\right). \end{aligned} \tag{2.8}$$

On taking limit as $k \longrightarrow \infty$ in above relation, we obtain that

$$\lim_{k\longrightarrow\infty} \omega_\lambda\left(x_{n_k}, x_{m_k}\right) = \varepsilon. \tag{2.9}$$

Also,

$$\begin{aligned}\omega_\lambda\left(x_{n_k}, x_{m_k}\right) &= \omega_{\max\{\lambda,\lambda\}}\left(x_{n_k}, x_{m_k}\right) \\ &\le \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right) + \omega_\lambda\left(x_{n_k+1}, x_{m_k}\right) \\ &= \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right) + \omega_{\max\{\lambda,\lambda\}}\left(x_{n_k+1}, x_{m_k}\right) \\ &\le \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right) + \omega_\lambda\left(x_{n_k+1}, x_{m_k+1}\right) + \omega_\lambda\left(x_{m_k+1}, x_{m_k}\right)\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}\omega_\lambda\left(x_{n_k+1}, x_{m_k+1}\right) &= \omega_{\max\{\lambda,\lambda\}}\left(x_{n_k+1}, x_{m_k+1}\right) \\ &\le \omega_\lambda\left(x_{n_k+1}, x_{n_k}\right) + \omega_\lambda\left(x_{n_k}, x_{m_k+1}\right) \\ &= \omega_\lambda\left(x_{n_k+1}, x_{n_k}\right) + \omega_{\max\{\lambda,\lambda\}}\left(x_{n_k}, x_{m_k+1}\right) \\ &\le \omega_\lambda\left(x_{n_k+1}, x_{n_k}\right) + \omega_\lambda\left(x_{n_k}, x_{m_k}\right) + \omega_\lambda\left(x_{m_k}, x_{m_k+1}\right).\end{aligned} \tag{2.11}$$

Using (2.7) and (2.9), by taking limit as $k \longrightarrow \infty$ in (2.10) and (2.11) we deduce that

$$\lim_{k\longrightarrow\infty} \omega_\lambda\left(x_{n_k+1}, x_{m_k+1}\right) = \varepsilon. \tag{2.12}$$

Moreover, from (2.7) and (iv) of Definition 1.7, it follows that

$$\begin{aligned}\omega_\lambda\left(x_{n_k}, x_{m_k+1}\right) &= \omega_{\max\{\lambda,\lambda\}}\left(x_{n_k}, x_{m_k+1}\right) \\ &\le \omega_\lambda\left(x_{n_k}, x_{m_k-1}\right) + \omega_\lambda\left(x_{m_k-1}, x_{m_k+1}\right) \\ &= \omega_\lambda\left(x_{n_k}, x_{m_k-1}\right) + \omega_{\max\{\lambda,\lambda\}}\left(x_{m_k-1}, x_{m_k+1}\right) \\ &\le \omega_\lambda\left(x_{n_k}, x_{m_k-1}\right) + \omega_\lambda\left(x_{m_k-1}, x_{m_k}\right) + \omega_\lambda\left(x_{m_k}, x_{m_k+1}\right) \\ &< \omega_\lambda\left(x_{m_k-1}, x_{m_k}\right) + \omega_\lambda\left(x_{m_k}, x_{m_k+1}\right) + \varepsilon.\end{aligned} \tag{2.13}$$

and

$$\begin{aligned}\omega_\lambda\left(x_{m_k}, x_{n_k+1}\right) &= \omega_{\max\{\lambda,\lambda\}}\left(x_{m_k}, x_{n_k+1}\right) \\ &\le \omega_\lambda\left(x_{m_k}, x_{n_k}\right) + \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right) \\ &= \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right) + \omega_{\max\{\lambda,\lambda\}}\left(x_{m_k}, x_{n_k}\right) \\ &\le \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right) + \omega_\lambda\left(x_{m_k}, x_{m_k-1}\right) + \omega_\lambda\left(x_{m_k-1}, x_{n_k}\right) \\ &< \varepsilon + \omega_\lambda\left(x_{m_k}, x_{m_k-1}\right) + \omega_\lambda\left(x_{n_k}, x_{n_k+1}\right).\end{aligned} \tag{2.14}$$

Moreover, since f is a $\alpha-$admissible we get $\alpha\left(x_{n_k}, x_{m_k}\right) \ge 1$. Regarding the fact f is a generalized $\alpha-(\psi,\phi)-Z_G-$contractive mapping, then by (2.1),

$$\begin{aligned}& C_G \\ &\le \zeta\left(\psi\left(\alpha\left(x_{n_k}, x_{m_k}\right)\omega_\lambda\left(fx_{n_k}, fx_{m_k}\right)\right), \psi\left(M\left(x_{n_k}, x_{m_k}\right)\right) - \phi\left(M\left(x_{n_k}, x_{m_k}\right)\right)\right) \\ &= \zeta\left(\psi\left(\alpha\left(x_{n_k}, x_{m_k}\right)\omega_\lambda\left(x_{n_k+1}, x_{m_k+1}\right)\right), \psi\left(M\left(x_{n_k}, x_{m_k}\right)\right) - \phi\left(M\left(x_{n_k}, x_{m_k}\right)\right)\right) \\ &< G\left(\psi\left(M\left(x_{n_k}, x_{m_k}\right)\right) - \phi\left(M\left(x_{n_k}, x_{m_k}\right)\right), \psi\left(\alpha\left(x_{n_k}, x_{m_k}\right)\omega_\lambda\left(x_{n_k+1}, x_{m_k+1}\right)\right)\right).\end{aligned} \tag{2.15}$$

Therefore by (i) of Definition 1.12, we get

$$\psi\left(\alpha\left(x_{n_k}, x_{m_k}\right)\omega_\lambda\left(x_{n_k+1}, x_{m_k+1}\right)\right) < \psi\left(M\left(x_{n_k}, x_{m_k}\right)\right) - \phi\left(M\left(x_{n_k}, x_{m_k}\right)\right), \tag{2.16}$$

where

$$M\left(x_{n_k},x_{m_k}\right)=\max\left\{\omega_\lambda\left(x_{n_k},x_{m_k}\right),\omega_\lambda\left(x_{n_k},fx_{n_k}\right),\omega_\lambda\left(x_{m_k},fx_{m_k}\right),\frac{\omega_\lambda\left(x_{n_k},fx_{m_k}\right)+\omega_\lambda\left(x_{m_k},fx_{n_k}\right)}{2}\right\}$$
$$=\max\left\{\omega_\lambda\left(x_{n_k},x_{m_k}\right),\omega_\lambda\left(x_{n_k},x_{n_k+1}\right),\omega_\lambda\left(x_{m_k},x_{m_k+1}\right),\frac{\omega_\lambda\left(x_{n_k},x_{m_k+1}\right)+\omega_\lambda\left(x_{m_k},x_{n_k+1}\right)}{2}\right\}.$$

Taking limit as $k\longrightarrow\infty$ in (2.16) and by using (2.4),(2.12), (2.13), (2.14), we have

$$\psi(\varepsilon)<\psi(\varepsilon)-\phi(\varepsilon)<\psi(\varepsilon), \tag{2.17}$$

which is a contradiction. Hence $\{x_n\}$ is a $\omega-$Cauchy sequence.

As X_ω is a $\omega-$complete non-Archimedean modular metric space, there exists $z\in X_\omega$ such that

$$\lim_{n\longrightarrow\infty}\omega_\lambda(x_n,z)=0. \tag{2.18}$$

Since f is $\omega-$continuous, we derive that

$$\lim_{n\longrightarrow\infty}\omega_\lambda(fx_n,fz)=\lim_{n\longrightarrow\infty}\omega_\lambda(x_{n+1},fz)=0. \tag{2.19}$$

From (2.18), (2.19) and the uniqueness of the limit, we conclude that z is a fixed point of f. □

Theorem 2.3. *Let X_ω be a complete non-Archimedean modular metric space and f be a generalized $\alpha-(\psi,\phi)-Z_G-$contractive mapping. Suppose that the following conditions hold:*

i. f is $\alpha-$admissible mapping,

ii. there exists $x_0\in X_\omega$ such that $\alpha(x_0,fx_0)\geq 1$,

iii. if $\{x_n\}$ is a sequence such that $\alpha(x_n,x_{n+1})\geq 1$ for all n and $x_n\longrightarrow x\in X_\omega$ as $n\longrightarrow\infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha\left(x_{n_k},x\right)\geq 1$ for all k.

Then, f has a fixed point.

Proof. Following the proof of Theorem 2.2, we know that the sequence $\{x_n\}$ defined by $x_{n+1}=fx_n$ for all $n\in\mathbb{N}$, converges for some $z\in X_\omega$. From (2.2) and condition (iii), there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha\left(x_{n_k},z\right)\geq 1$ for all k. Next, we will show that z is a fixed point of f. We assume that $z\neq fz$, that is $\omega_\lambda(z,fz)>0$. By f is a generalized $\alpha-(\psi,\phi)-Z_G-$contractive mapping, we have

$$\begin{aligned}C_G&\leq\zeta\left(\psi\left(\alpha\left(x_{n_k},z\right)\omega_\lambda\left(fx_{n_k},fz\right)\right),\psi\left(M\left(x_{n_k},z\right)\right)-\phi\left(M\left(x_{n_k},z\right)\right)\right)\\&=\zeta\left(\psi\left(\alpha\left(x_{n_k},z\right)\omega_\lambda\left(x_{n_k+1},fz\right)\right),\psi\left(M\left(x_{n_k},z\right)\right)-\phi\left(M\left(x_{n_k},z\right)\right)\right)\\&<G\left(\psi\left(M\left(x_{n_k},z\right)\right)-\phi\left(M\left(x_{n_k},z\right)\right),\psi\left(\alpha\left(x_{n_k},z\right)\omega_\lambda\left(x_{n_k+1},fz\right)\right)\right).\end{aligned} \tag{2.20}$$

and then by (i) of the Definition 1.12, we get

$$\psi\left(\alpha\left(x_{n_k},z\right)\omega_\lambda\left(x_{n_k+1},fz\right)\right)<\psi\left(M\left(x_{n_k},z\right)\right)-\phi\left(M\left(x_{n_k},z\right)\right) \tag{2.21}$$

which is equivalent to

$$\begin{aligned}\psi\left(\omega_\lambda\left(x_{n_k+1}, fz\right)\right) &\leq \psi\left(\alpha\left(x_{n_k}, z\right)\omega_\lambda\left(x_{n_k+1}, fz\right)\right)\\ &< \psi\left(M\left(x_{n_k}, z\right)\right) - \phi\left(M\left(x_{n_k}, z\right)\right),\end{aligned} \tag{2.22}$$

where

$$\begin{aligned}M\left(x_n, z\right) &= \max\left\{\omega_\lambda\left(x_n, z\right), \omega_\lambda\left(x_n, fx_n\right), \omega_\lambda\left(z, fz\right), \tfrac{\omega_\lambda\left(x_n, fz\right)+\omega_\lambda\left(z, fx_n\right)}{2}\right\}\\ &= \max\left\{\omega_\lambda\left(x_n, z\right), \omega_\lambda\left(x_n, x_{n+1}\right), \omega_\lambda\left(z, fz\right), \tfrac{\omega_\lambda\left(x_n, fz\right)+\omega_\lambda\left(z, x_{n+1}\right)}{2}\right\}.\end{aligned} \tag{2.23}$$

Taking limit as $n \longrightarrow \infty$ in (2.22), by (2.23) we get

$$\psi\left(\omega_\lambda\left(z, fz\right)\right) < \psi\left(\omega_\lambda\left(z, fz\right)\right) - \phi\left(\omega_\lambda\left(z, fz\right)\right) < \psi\left(\omega_\lambda\left(z, fz\right)\right)$$

which is a contradiction. Thus $z = fz$ and hence z is a fixed point f. □

For the uniqueness of a fixed point of f, we shall suggest the following hypothesis.

(U) For all $x, y \in \mathrm{Fix}\left(\mathrm{f}\right)$, which denotes the set of fixed points of f, we get $\alpha\left(x, y\right) \geq 1$.

Theorem 2.4. *Adding condition (U) to the hypotheses of Theorem 2.2 (resp. Theorem 2.3), we obtain that f has a unique fixed point.*

Proof. We assume that u is an another fixed point, that is, $\omega_\lambda\left(z, u\right) \neq 0$. From hypothesis, we get $\alpha\left(z, u\right) \geq 1$. By using (2.1), we have

$$\psi\left(\alpha\left(z, u\right)\omega_\lambda\left(z, u\right)\right) \tag{2.24}$$

and from (i) of the Definition 1.12, we get

$$\psi\left(\omega_\lambda\left(z, u\right)\right) \leq \psi\left(\alpha\left(z, u\right)\omega_\lambda\left(z, u\right)\right) < \psi\left(M\left(z, u\right)\right) - \phi\left(M\left(z, u\right)\right) \tag{2.25}$$

where

$$\begin{aligned}M\left(z, u\right) &= \max\left\{\omega_\lambda\left(z, u\right), \omega_\lambda\left(z, fz\right), \omega_\lambda\left(u, fu\right), \tfrac{\omega_\lambda\left(z, fu\right)+\omega_\lambda\left(u, fz\right)}{2}\right\}\\ &= \max\left\{\omega_\lambda\left(z, u\right), 0, 0, \tfrac{\omega_\lambda\left(z, u\right)+\omega_\lambda\left(z, u\right)}{2}\right\}\\ &= \omega_\lambda\left(z, u\right).\end{aligned} \tag{2.26}$$

From (2.25) and (2.26), we deduce that

$$\psi\left(\omega_\lambda\left(z, u\right)\right) < \psi\left(\omega_\lambda\left(z, u\right)\right) - \phi\left(\omega_\lambda\left(z, u\right)\right) < \psi\left(\omega_\lambda\left(z, u\right)\right)$$

which is a contradiction. Hence $z = u$. □

Example 2.5. *Let $X_\omega = \mathbb{R}$, $\omega_\lambda\left(x, y\right) = \frac{1}{\lambda}\left|x - y\right|$ for all $x, y \in X_\omega$. Define the mappings $f : X_\omega \to X_\omega$ as follow:*

$$fx = \frac{x}{5}, \quad x \in X_\omega,$$

and $\alpha : X_\omega \times X_\omega \longrightarrow [0, +\infty)$ defined by

$$\alpha\left(x, y\right) = \begin{cases} 1, & x, y \in [0, 1] \\ 0, & \text{otherwise}. \end{cases}$$

If we consider the functions $\zeta\left(t, s\right) = \frac{3}{4}s - t$, $G\left(s, t\right) = s - t$ and $C_G = 0$, $\psi\left(t\right) = t$ and $\phi\left(t\right) = \frac{t}{2}$, then all the hypotheses of Theorem 2.2-2.4 are satisfied and $x = 0$ is a unique fixed point of f.

Now, we modify the contractive condition and introduce rational type generalized weak contraction using simulation function and $C-$class function as follows.

Definition 2.6. *Let X_ω be a non-Archimedean modular metric space and $f : X_\omega \longrightarrow X_\omega$ and $\alpha : X_\omega \times X_\omega \longrightarrow [0,\infty)$ be mappings. We say that f is rational type generalized $\alpha-$simulation contractive mapping if there exists $\zeta \in Z_G$ such that*

$$\zeta\left(\psi\left(\alpha\left(x,y\right)\omega_\lambda\left(fx,fy\right)\right),\psi\left(m\left(x,y\right)\right)-\phi\left(m\left(x,y\right)\right)\right) \geq C_G, \tag{2.27}$$

where

$$m\left(x,y\right)=\max\left\{\omega_\lambda\left(x,y\right),\omega_\lambda\left(x,fx\right),\omega_\lambda\left(y,fy\right),\right.$$

$$\left.\frac{\omega_\lambda(x,fx)\omega_\lambda(y,fy)}{1+\omega_\lambda(fx,fy)},\frac{\omega_\lambda(x,fx)\omega_\lambda(y,fy)}{1+\omega_\lambda(x,y)}\right\},$$

$\psi \in \Psi$, $\phi \in \Phi$, for all $x,y \in X_\omega$.

Theorem 2.7. *Let X_ω be a complete non-Archimedean modular metric space and f be a rational type generalized $\alpha-$simulation contractive mapping. Suppose that the following conditions hold:*

i. f is $\alpha-$admissible mapping,

ii. there exists $x_0 \in X_\omega$ such that $\alpha\left(x_0,fx_0\right) \geq 1$,

iii. f is $\omega-$continuous.

Then, f has a fixed point.

Proof. Let $x_0 \in X_\omega$ be such that $\alpha\left(x_0,fx_0\right) \geq 1$. Define an iterative sequence $\{x_n\}$ in X by letting $x_{n+1} = fx_n$ for all $n \in \mathbb{N}$. From (2.27) and (2.2),

$$\begin{aligned}
& C_G \\
& \leq \zeta\left(\psi\left(\alpha\left(x_n,x_{n-1}\right)\omega_\lambda\left(fx_n,fx_{n-1}\right)\right),\psi\left(m\left(x_n,x_{n-1}\right)\right)-\phi\left(m\left(x_n,x_{n-1}\right)\right)\right) \\
& = \zeta\left(\psi\left(\alpha\left(x_n,x_{n-1}\right)\omega_\lambda\left(x_{n+1},x_n\right)\right),\psi\left(m\left(x_n,x_{n-1}\right)\right)-\phi\left(m\left(x_n,x_{n-1}\right)\right)\right) \\
& < G\left(\psi\left(m\left(x_n,x_{n-1}\right)\right)-\phi\left(m\left(x_n,x_{n-1}\right)\right),\psi\left(\alpha\left(x_n,x_{n-1}\right)\omega_\lambda\left(x_{n+1},x_n\right)\right)\right).
\end{aligned}$$

Further, using (i) of Definition 1.12, we have

$$\psi\left(\alpha\left(x_n,x_{n-1}\right)\omega_\lambda\left(x_{n+1},x_n\right)\right) < \psi\left(m\left(x_n,x_{n-1}\right)\right)-\phi\left(m\left(x_n,x_{n-1}\right)\right). \tag{2.28}$$

Consequently, we get that

$$\begin{aligned}
\psi\left(\omega_\lambda\left(x_{n+1},x_n\right)\right) & \leq \psi\left(\alpha\left(x_n,x_{n-1}\right)\omega_\lambda\left(x_{n+1},x_n\right)\right) \\
& < \psi\left(m\left(x_n,x_{n-1}\right)\right)-\phi\left(m\left(x_n,x_{n-1}\right)\right) \\
& < \psi\left(m\left(x_n,x_{n-1}\right)\right),
\end{aligned} \tag{2.29}$$

where,

$$m(x_n, x_{n-1}) = \max\{\omega_\lambda(x_n, x_{n-1}), \omega_\lambda(x_n, fx_n), \omega_\lambda(x_{n-1}, fx_{n-1}), \frac{\omega_\lambda(x_n, fx_n)\,\omega_\lambda(x_{n-1}, fx_{n-1})}{1+\omega_\lambda(fx_n, fx_{n-1})}, \frac{\omega_\lambda(x_n, fx_n)\,\omega_\lambda(x_{n-1}, fx_{n-1})}{1+\omega_\lambda(x_n, x_{n-1})}\}$$

$$= \max\{\omega_\lambda(x_n, x_{n-1}), \omega_\lambda(x_n, x_{n+1}), \omega_\lambda(x_{n-1}, x_n), \frac{\omega_\lambda(x_n, x_{n+1})\,\omega_\lambda(x_{n-1}, x_n)}{1+\omega_\lambda(x_n, x_{n+1})}, \frac{\omega_\lambda(x_n, x_{n+1})\,\omega_\lambda(x_{n-1}, x_n)}{1+\omega_\lambda(x_{n-1}, x_n)}\}$$

$$= \max\{\omega_\lambda(x_{n-1}, x_n), \omega_\lambda(x_n, x_{n+1})\}.$$

Now if $\max\{\omega_\lambda(x_{n-1}, x_n), \omega_\lambda(x_n, x_{n+1})\} = \omega_\lambda(x_n, x_{n+1})$, then from (2.29) we have a contradiction. So, $m(x_n, x_{n-1}) = \omega_\lambda(x_n, x_{n-1})$ for all $n \in \mathbb{N}$. Consequently, from (2.29) and since ψ is nondecreasing, we gives

$$\omega_\lambda(x_{n+1}, x_n) < \omega_\lambda(x_n, x_{n-1}), \tag{2.30}$$

for all $n \in \mathbb{N}$. So $\{\omega_\lambda(x_{n+1}, x_n)\}$ is a monotonically decreasing sequence of nonnegative real numbers, and hence there exists $r \geq 0$ such that

$$\lim_{n\longrightarrow\infty} \omega_\lambda(x_{n+1}, x_n) = r. \tag{2.31}$$

Assume that $r > 0$. Then using rational type generalized $\alpha-$simulation contractive mapping of f and (b) of the Definition 1.11, we have

$$\begin{aligned} C_G &\leq \limsup_{n\longrightarrow\infty} \zeta(\psi(\alpha(x_n, x_{n-1})\,\omega_\lambda(fx_n, fx_{n-1})), \\ &\psi(m(x_n, x_{n-1})) - \phi(m(x_n, x_{n-1}))) \\ &\leq \limsup_{n\longrightarrow\infty} \zeta(\psi(\alpha(x_n, x_{n-1})\,\omega_\lambda(x_{n+1}, x_n)), \\ &\psi(\omega_\lambda(x_n, x_{n-1})) - \phi(\omega_\lambda(x_n, x_{n-1}))) \\ &\leq \limsup_{n\longrightarrow\infty} \zeta(\psi(r), \psi(r)) \\ &< C_G, \end{aligned}$$

which is a contradiction and so $r = 0$. Also, using the same technique as in Theorem 2.2, we have $\{x_n\}$ is a $\omega-$Cauchy sequence.

As X_ω is a $\omega-$complete non-Archimedean modular metric space, there exists $z \in X_\omega$ such that

$$\lim_{n\longrightarrow\infty} \omega_\lambda(x_n, z) = 0. \tag{2.32}$$

Since f is $\omega-$continuous, we derive that

$$\lim_{n\longrightarrow\infty} \omega_\lambda(fx_n, fz) = \lim_{n\longrightarrow\infty} \omega_\lambda(x_{n+1}, fz) = 0. \tag{2.33}$$

From (2.32), (2.33) and the uniqueness of the limit, we conclude that z is a fixed point of f. □

3 Consequences

In this section, we will illustrate that non-Archimedean modular metric version of several existing fixed point results in the literature can be derived from our main results.

Corollary 3.1. *Let X_ω be a complete non-Archimedean modular metric space. Suppose that the following conditions hold:*

i. f *is* $\alpha-$*admissible mapping,*

ii. *there exists* $x_0 \in X_\omega$ *such that* $\alpha(x_0, fx_0) \geq 1$,

iii. f *is* $\omega-$*continuous or if* $\{x_n\}$ *is a sequence such that* $\alpha(x_n, x_{n+1}) \geq 1$ *for all* n *and* $x_n \longrightarrow x \in X_\omega$ *as* $n \longrightarrow \infty$, *then there exists a subsequence* $\{x_{n_k}\}$ *of* $\{x_n\}$ *such that* $\alpha(x_{n_k}, x) \geq 1$ *for all* k,

iv. *there exists a* $\zeta \in Z_G$ *such that for all* $x, y \in X_\omega$

$$\zeta(\alpha(x,y)\,\omega_\lambda(fx, fy), M(x,y) - \phi(M(x,y))) \geq C_G,$$

$M(x,y) = \max\left\{\omega_\lambda(x,y), \omega_\lambda(x,fx), \omega_\lambda(y,fy), \frac{\omega_\lambda(x,fy)+\omega_\lambda(y,fx)}{2}\right\}$ *and* $\phi \in \Phi$.

Then, f *has a fixed point.*

Corollary 3.2. *Let X_ω be a complete non-Archimedean modular metric space. Suppose that the following conditions hold:*

i. f *is* $\alpha-$*admissible mapping,*

ii. *there exists* $x_0 \in X_\omega$ *such that* $\alpha(x_0, fx_0) \geq 1$,

iii. f *is* $\omega-$*continuous or if* $\{x_n\}$ *is a sequence such that* $\alpha(x_n, x_{n+1}) \geq 1$ *for all* n *and* $x_n \longrightarrow x \in X_\omega$ *as* $n \longrightarrow \infty$, *then there exists a subsequence* $\{x_{n_k}\}$ *of* $\{x_n\}$ *such that* $\alpha(x_{n_k}, x) \geq 1$ *for all* k,

iv. *there exists a* $\zeta \in Z_G$ *such that for all* $x, y \in X_\omega$

$$\zeta(\alpha(x,y)\,\omega_\lambda(fx, fy), \omega_\lambda(x,y) - \phi(\omega_\lambda(x,y))) \geq C_G,$$

and $\phi \in \Phi$.

Then, f *has a fixed point.*

Corollary 3.3. *Let X_ω be a complete non-Archimedean modular metric space. Suppose that the following conditions hold:*

i. f *is* $\alpha-$*admissible mapping,*

ii. *there exists* $x_0 \in X_\omega$ *such that* $\alpha(x_0, fx_0) \geq 1$,

iii. f *is* $\omega-$*continuous or if* $\{x_n\}$ *is a sequence such that* $\alpha(x_n, x_{n+1}) \geq 1$ *for all* n *and* $x_n \longrightarrow x \in X_\omega$ *as* $n \longrightarrow \infty$, *then there exists a subsequence* $\{x_{n_k}\}$ *of* $\{x_n\}$ *such that* $\alpha(x_{n_k}, x) \geq 1$ *for all* k, *iv. there exist $\zeta \in Z_G$ and $k \in (0,1)$ such that for all $x,y \in X_\omega$ and*

$$\zeta\left(\alpha\left(x,y\right)\omega_\lambda\left(fx,fy\right),kM\left(x,y\right)\right)\geq C_G,$$

$$M\left(x,y\right)=\max\left\{\omega_\lambda\left(x,y\right),\omega_\lambda\left(x,fx\right),\omega_\lambda\left(y,fy\right),\frac{\omega_\lambda(x,fy)+\omega_\lambda(y,fx)}{2}\right\} \text{ and } \phi\in\Phi.$$

Then, f has a fixed point.

Proof. It suffices to take $\psi(t)=t$ and $\phi(t)=1-kt,\ k\in(0,1)$ in main results. □

Remark 3.4. *If we take $\alpha(x,y)=1$ for all $x,y\in X_\omega$ in Corollary 3.2 and 3.3, we obtain that non-Archimedean modular metric version of Simulation type weak-contraction and Ciric-contraction which are well-known fixed point theorems in the literature.*

Remark 3.5. *In the above corollaries, If we take $m(x,y)$ instead of $M(x,y)$ we obtain other fixed point results.*

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