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RESEARCH ARTICLE

Existence and stability results for stochastic functional integro-differential equations with poisson jumps under non-Lipschitz conditions

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Abstract: In this paper, we investigate the existence results on averaging principle and stability of mild solutions for stochastic functional integro-differential equations with poisson jumps under non-Lipschitz condition. We establish the results by the method of successive approximation and Biharis inequality under the theory of resolvent operators.

Keywords: Existence, Uniqueness, Stability, Averaging principle, Biharis inequality, Resolvent operator, Successive Approximation

MSC: 34K50, 34C29,60H15.

1 Introduction

The averaging principle is to approximate the solutions of a more complicated time varying differential system with the solutions of an autonomous differential system under some suitable conditions. It is effective for exploring stochastic differential equations in mechanics, physics, control and many other areas. Less amount of work has been done in the field of averaging principle. The rigorous results on averaging principles were firstly put forward by Krylov and Bogolyubov [**KN**]. After that, Khasminshii [**K, K1**]

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considered averaging principles of Ito stochastic differential equations, parabolic and elliptic differential equations. Recently, few authors studied the averaging principle for stochastic differential equation under some restrictive conditions with non-Lipschitz conditions Mao et al. [**MY**]; Tan and Lei [**TL**]. In [**XD**], Duan and Xu established averaging principle for dynamical systems with Levy noise. Likewise stochastic differential equation, stochastic integro-differential equation also has wide applications in the areas of mechanics, electrical engineering and so on [see Liu and Ezzinbi [**LI**]; Ding, Liang and Xiao [**DL**] and the references therein]. Anguraj and Vinodkumar [**AA**] gave the results on impulsive stochastic neutral functional differential equations with non-Lipschitz condition.

On the other hand, most systems in science and industry are perturbed by some random environmental effects, which are often described as Gaussian noise. As a special non-Gaussian Levy noise, poisson noise is usually a hot spot when dealing with stochastic systems. Stoyanov and Bainov [**IM**] investigated the averaging method for a class of stochastic differential equations with poisson Jumps. A more natural mathematical framework for these phenomena has been taken into account other than purely Brownian perturbations. It is recognized that stochastic differential equations with jumps are quite suitable to describe such discontinuous systems. In the meantime, the averaging method for a class of stochastic differential equations with jumps has been received much attention for many authors and there exists some literatures [**IM**, **VG**, **XU**] concerned with the averaging method for stochastic differential equations with jumps.

The study of averaging principle for stochastic integro-differential systems with resolvent operator and phase axioms is an unprocessed issue and it is also the motivation of this paper. To the best of our knoeledge, there is no work reported on the averaging principle and stability for the stochastic integro-differential equations with resolvent operators under non-Lipschitz case . The purpose of this paper is to fill the gap in the study of averaging principle and stability for abstract stochastic functional integro-differential equations with poisson jumps under non-Lipschith conditions.

$$\begin{aligned} d[x(t)] &= A(t)\left[x(t)+\int_0^t f(t,s)x(s)ds\right]dt + g(t,x_t)dt + \sigma(t,x_t)dw(t) \\ &+ \int_Z h(t,x_t,u)\tilde{N}(dt,du), \;\; t\in J=[0,T] \\ x_0 &= \varphi \in \mathcal{B}. \end{aligned} \tag{1}$$

This paper is organised as follows. In section 2, we give some basic definitions, lemmas and notations which will be used in our results. We give the proof of the existence, uniqueness, averaging principle and stability results for problem (1), respectively, in the sections 3 and 4.

2 Preliminaries

Let $\mathbb{X},\mathbb{Y}$ be real separable Hilbert spaces and $L(\mathbb{Y},\mathbb{X})$ be the space of bounded linear operators mapping $\mathbb{Y}$ into $\mathbb{X}$. Let $(\Omega,\Im,P;\mathbb{F})$ $(\mathbb{F}=\{\Im_t\}_{t\geq 0})$ be a complete filtered probability space such that $\Im_0$ contains all P-null sets of $\Im$. An $\mathbb{X}$-valued random variable is an $\Im$-measurable function $x(t):\Omega\to\mathbb{X}$ and the collection of random variables $\mathcal{H} = \{x(t,\omega) : \Omega \to \mathbb{X} : t \in J\}$ is called a stochastic process. In general, we just write $x(t)$ instead of $x(t,\omega)$ and $x(t) : J \to \mathbb{X}$ is the space of $\mathcal{H}$. Let $\{e_i\}_{i\leq 1}$ be a complete orthonormal basis of $\mathbb{Y}$. Suppose that $\{\mathcal{W}(t) : t \leq 0\}$ is a cylindrical $\mathbb{Y}$-valued Wiener process with finite trace nuclear covariance operator $Q \geq 0$, denote $Tr(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies $Qe_i = \lambda_i e_i$. Actually,

$$\mathcal{W}(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i} w_i(t) e_i,$$

where $\{w_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\Im_t = \sigma\{w(s) : 0 \leq s \leq t\}$ is the σ-algebra generated by $\mathcal{W}$ and $\Im_t = \Im$. Let $\mu \in (\mathbb{Y},\mathbb{X})$ and define

$$\|\mu\|_Q^2 \quad = \quad Tr(\mu Q \mu^*) = \sum_{n=1}^{\infty} \left\| \sqrt{\lambda_n} \mu e_n \right\|^2.$$

If $\|\mu\|_Q < \infty$, then μ is called a Q-Hilbert-Schmidt operator. Let $L_Q(\mathbb{Y},\mathbb{X})$ denotes the space of all Q-Hilbert-Schmit operator $\mu : \mathbb{Y} \to \mathbb{X}$. The completion $L_Q(\mathbb{Y},\mathbb{X})$ of $L(\mathbb{Y},\mathbb{X})$ with respect to the topology induced by the norm $\|\cdot\|_Q$, where $\|\mu\|_Q^2 = \langle \mu, \mu \rangle$ is a Hilbert space with the above norm topology.

Let $(Z, \xi, v(du))$ be a σ-finite measurable space. Given a stationary poisson point process $(p_t)_{t\geq 0}$, which is defined on $(\Omega, \Im, P; \mathbb{F})$ with values in Z and with characteristic measure v. We will denote by $\mathcal{N}(t, du)$ be the counting measure of p_t such that $\tilde{\mathcal{N}}(t, A) = E(\mathcal{N}(t, A)) = tv(A)$ for $A \in \xi$. Dfine

$$\tilde{\mathcal{N}}(t, du) = \mathcal{N}(t, du) - tv(du)$$

the poisson martingle measure generated by p_t.

We now make the system (1) precise. The family of operators $A(t) : 0 \leq t \leq T$ is the infinitesimal generator of a strongly continuous semigroup on $\mathbb{X}$, which is independent of t, and $f(t,s), t, s \in J$ is a bounded linear operator. Assume that $g : \mathbb{R}^+ \times \mathcal{B} \to \mathbb{X}$, $\sigma : \mathbb{R}^+ \times \mathcal{B} \to L_Q(\mathbb{Y},\mathbb{X})$, and $h : \mathbb{R}^+ \times \mathcal{B} \times Z \to \mathbb{X}$, are Borel measurable and the phase space $\mathcal{B}$, which will be described axiomatically below.

We utilise the axiomatic definition of the phase space $\mathcal{B}$ introduced by Hale and Kato **[HK]** and Hinto, Murakami and Naito **[HM]**. The axioms of the space $\mathcal{B}$ are established for $\mathcal{F}_0$-measurable functions from $(-\infty, 0]$ into $\mathbb{X}$, endowed with a norm $\|\cdot\|_{\mathcal{B}}$ is given as the following axioms:

(A1) If $x : (-\infty, T] \to \mathbb{X}, T > 0$ is such that $x_0 \in \mathcal{B}$ then for every $t \in [0, T]$, the following conditions hold:

1. $x_t \in \mathcal{B}$,

2. $\|x(t)\| \leq L \|x_t\|_{\mathcal{B}}$,

3. $\|x_t\|_{\mathcal{B}} \leq M(t) \sup_{0\leq s\leq t} \|x(s) + N(t)x_0\|_{\mathcal{B}}$, where $L > 0$ is constant; $M(\cdot)$, $N(\cdot) : [0, +\infty) \to [1, +\infty)$, is continuous $N(\cdot)$ is locally bounded, and $L, M(\cdot), N(\cdot)$ are independent of $x(\cdot)$.

(A2) For the function $x(\cdot)$ in $(A1)$, x_t is a $\mathcal{B}$-valued continuous function on $[0, T]$ and the space $\mathcal{B}$ is complete.

Let us denote $L_2(\Omega, \mathcal{F}, P; \mathcal{B}) = L_2(\Omega; \mathcal{B})$ be the collection of all strongly measurable, square integrable, $\mathcal{B}$-valued random variables. Where $L_2(\Omega; \mathcal{B})$ is the Banach space equipped with norm $\|x(\cdot)\|_{L_2}^2 = E\,\|x(\cdot, w)\|_{\mathbb{X}}^2$. Here the expectation is defined by $E_\chi = \int_\Omega \chi(w)dP$. We denote by $L_2^{\mathcal{F}_0}(\Omega, \mathcal{B})$ the family of all almost surely bounded, continuous, $\mathcal{F}_0$-measurable, $\mathcal{B}$-valued random variables. Let $\mathcal{B}_{\mathcal{T}}$ be the Banach space of all $\mathcal{F}_t$-adapted process $x(t, w)$ which is almost surely continuous in t for fixed $w \in \Omega$ with norm

$$\|x\|_{\mathcal{B}} = \left(\sup_{t \in J} \|x_t\|_{\mathcal{B}}^2\right)^{\frac{1}{2}},$$

where

$$\|x_t\|_{\mathcal{B}} \leq N_T E\,\|\varphi\|_{\mathcal{B}} + M_T \sup\{E\,\|x(s)\| : 0 \leq s \leq T\},$$

$M_T = \sup_{t \in J}\{M(t)\}$, $N_T = \sup_{t \in J}\{N(t)\}$. We shell assume throughout the remainder of the paper that the initial function $\varphi \in L_2^{\mathcal{F}_0}(\Omega, \mathcal{B})$

The resolvent operator plays an important role in the study of the existence of solutions for partial integro-differential equations and to give a variation of constant formula for linear systems. For more details on resolvent operator, reader may refer Liu and Ezzinbi **[LI]**; Grimmer **[G1]**.

Definition 2.1. *A resolvent operator for (1) is a bounded operator valued function $R(t, s) \in B(\mathbb{X})$, $0 \leq s \leq t \leq T$, the space of bounded linear operators on X is having the following properties*

1. *$R(t, s)$ is strongly continuous in s and t. $R(t, t) = I$, the identity operator on X. $\|R(t, s)\| \leq M$ for $t, s \in J$ and M is a constant.*

2. *$R(t, s)X \subset X$, $R(t, s)$ is strongly continuous in s and t on X.*

3. *For $y \in X$, $R(t, s)y$ is continuously differentiable in s and t, and for $0 \leq s \leq t \leq T$,*
$$\frac{\partial R(t, s)x}{\partial t} = A(t)\left[R(t, s)x + \int_s^t f(t, \tau)R(\tau, s)x d\tau\right].$$

Lemma 2.1. *(Bihari's inequality [**B**]) Let $T > 0$ and $u_0 \geq 0$, $u(t), v(t)$ be the continuous functions on $[0, T]$. Let $K : \mathbb{R}^+ \to \mathbb{R}^+$ be a concave continuous and nondecreasing function such that $K(r) > 0$ for $r > 0$. If*

$$u(t) \leq u_0 + \int_0^t v(s)K(u(s))ds \quad for \;\; all \; 0 \leq t \leq T$$

then

$$u(t) \leq G^{-1}\left(G(u_0) + \int_0^t v(s)ds\right) \quad for \;\; all \; t \in [0, T]$$

such that

$$G(u_0) + \int_0^t v(s)ds \in Dom(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$ *for* $r \geq 0$ *and* G^{-1} *is the inverse funciton of* G. *In particular, moreover if,* u_0 *and* $\int_{0^+} \frac{ds}{K(s)} = \infty$, *then* $u(t) = 0$ *for all* $t \in [0, T]$.

Lemma 2.2. *[**RE1**] Let the assumptions of Lemma 1 holds. If*

$$u(t) \leq u_0 + \int_t^T v(s)K(u(s))ds \ \ for \ \ all \ 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\left(G(u_0) + \int_t^T v(s)ds\right) \ \ for \ \ all \ t \in [0, T]$$

such that

$$G(u_0) + \int_t^T v(s)ds \in Dom(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$ *for* $r \geq 0$ *and* G^{-1} *is the inverse funciton of* G.

Corollary 2.1. *[**RE1**] Let the assumptions of Lemma 1 hold and* $v(t) \geq 0$ *for* $t \in [0, T]$. *If for all* $\epsilon > 0$, *there exists* $t_1 \geq 0$ *such that for* $0 \leq u_0 \leq \epsilon$, $\int_{t_1}^T v(s)ds \leq \int_{u_0}^{\epsilon} \frac{ds}{K(s)}$ *holds, then for every* $t \in [t_1, T]$, *the estimate* $u(t) \leq \epsilon$ *holds.*

Definition 2.2. *A stochastic process* $\{x(t) \in \mathcal{B}_{\mathcal{T}}, -\infty \leq t \leq T\}$ *is said to be a mild solution of the equations (1), if*

1. $x(t) \in X$ *is* $\mathcal{F}_t$*-adapted,*

2. *for each* $t \in (-\infty, T]$,$x(t)$ *satisfies the following integral equation*

$$\begin{aligned} x(t) &= R(t,0)\varphi(0) + \int_0^t R(t,s)g(s,x_s)ds + \int_0^t R(t,s)\sigma(s,x_s)dw(s) \\ &+ \int_0^t \int_Z R(t,s)h(s,x_s,u)\tilde{N}(ds,du) \ \ for \ \ a.s \ \ t \in [0,T] \\ x(t) &= \varphi(t), \ \ for \ \ t \in (-\infty, 0]. \end{aligned} \tag{2}$$

3 Existence, uniqueness and averaging principle

In this section, we examine the results of the existence, uniqueness and averaging principle of mild solution for the system (1). we test the following hypotheses to prove our results.

(H1). A is the infinitesimal generator of a strongly continuous semigroup $R(t,s)$, whose domain $D(A)$ is dense in $\mathbb{X}$ such that $\|R(t,s)\|^2 \leq M_1$, for all $t, s \in J = [0, T]$.

(H2). For all $t \in [0, T]$ and for each $u, v \in \mathcal{B}$ such that

$$\|g(t,u) - g(t,v)\|^2 \vee \|\sigma(t,u) - \sigma(t,v)\|^2 \leq \eta((\|u - v\|^2_{\mathcal{B}}),$$

where $\eta(\cdot)$ is a concave non-decreasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$, $\eta(0) = 0$, $\eta(u) > 0$, for $u > 0$

and $\int_0^+ \frac{du}{\eta(u)} = \infty$.

(H3). For all $t \in [0,T]$, it follows that $g(t,0)$,$\sigma(t,0) \in L^2$ such that

$$\|g(t,0)\|^2 \vee \|\sigma(t,0)\|^2 \vee \|h(t,0,u)\|^2 \leq \eta_0,$$

where $\eta_0 > 0$ is a constant.

(H4). For all $t \in [0,T]$, and for each $u, v \in \mathcal{B}$ such that

$$\begin{aligned}
(i). \int_0^t \int_Z \|h(t,x,u) - h(t,y,u)\|^2 v(du)ds \vee \\
\left(\int_0^t \int_Z \|h(t,x,u) - h(t,y,u)\|^4 v(du)ds\right)^{\frac{1}{2}} &\leq \left(\eta(\|u - v\|^2_{\mathcal{B}}\right), \\
(ii). \left(\int_0^t \int_Z \|h(t,x,u)\|^4 v(du)ds\right)^{\frac{1}{2}} &\leq \int_0^t \eta \, \|x\|^2_{\mathcal{B}} \, ds.
\end{aligned}$$

Let us now introduce the successive approximation to equation (2) as follows

$$\begin{aligned}
x^0(t) &= \varphi(t) \quad \text{for } \ t \in (-\infty, 0], \\
x^0(t) &= R(t,0)\varphi(0) \quad \text{for } \ t \in [0,T]
\end{aligned} \tag{3}$$

and, for $n = 1, 2, ...,$

$$\begin{aligned}
x^n(t) = \varphi(t) \ \text{ for } \ t \in (-\infty, 0], \\
x^n(t) = R(t,0)\varphi(0) + \int_0^t R(t,s)g(s, x_s^{n-1})ds \\
+ \int_0^t R(t,s)\sigma(s, x_s^{n-1})dw(s) + \int_0^t \int_Z h(s, x_s^{n-1}, u)\tilde{N}(ds, du)
\end{aligned} \tag{4}$$

for a.s., $t \in J$ with an arbitrary non-negative initial approximation $x^0 \in L_2(\Omega, \mathcal{B})$.

Theorem 3.1. *Suppose that $(H1) - (H4)$ holds, then the system (1) has unique mild solution $x(t)$ in $L_2(\Omega, \mathcal{B})$.*

Proof. Let $x^0 \in L_2(\Omega, \mathcal{B})$ be a fixed initial approximation to (4). Then for any $n \geq 1$, we have

$$\begin{aligned}
& E \, \|x^n(t)\|^2 \\
&\leq 4M_1 \, \|\varphi(0)\|^2 \\
&+ 8M_1 T \int_0^t \left[\left\| g(s, x_s^{n-1}) - g(s,0) \right\|^2 + \|g(s,0)\|^2 \right] ds \\
&+ 8M_1 \int_0^t \left[\left\| \sigma(s, x_s^{n-1}) - \sigma(s,0) \right\|^2 + \|\sigma(s,0)\|^2 \right] ds \\
&+ 8M_1 \int_0^t \int_Z \left[\left\| h(s, x_s^{n-1}, u) - h(s,0,u) \right\|^2 + \|h(s,0,u)\|^2 \right] v(du)ds \\
&+ 4M_1 \left(\int_0^t \int_Z \left[\left\| h(s, x_s^{n-1}, u) \right\|^4 \right] v(du)ds \right)^{\frac{1}{2}}.
\end{aligned}$$

Thus

$$E\|x_t^n\|_{\mathcal{B}}^2 \le \tau_1 + 4M_1(2T+5)E\int_0^t \eta(\left\|x_s^{n-1}\right\|_{\mathcal{B}}^2)ds.$$

where $\tau_1 = 4M_1[E\|\varphi(0)\|^2 + 2T(T+2)\eta_0]$.

Given that $\eta(\cdot)$ is concave and $\eta(0) = 0$, we can find a pair of positive constants a and b such that

$$\eta(u) \le a + bu, \text{ for all } u \le 0.$$

Then, we have

$$\begin{aligned} E\|x_t^n\|_{\mathcal{B}}^2 &\le \tau_2 + 4M_1(2T+5)b\int_0^t E\left\|x_s^{n-1}\right\|_{\mathcal{B}}^2 ds \\ &\le \tau_2 + 4M_1(2T+5)b\int_0^t [N(t)E\|\varphi\|_{\mathcal{B}}^2 + M(t)\sup_{0\le s\le T} E\left\|x^{n-1}(s)\right\|^2]ds \\ &\le \tau_2 + 4M_1T(2T+5)bN_T E\|\varphi\|_{\mathcal{B}}^2 \\ &+ 4M_1M_T(2T+5)b\int_0^t \sup_{0\le s\le T} E\left\|x^{n-1}(s)\right\|^2 ds, \quad n = 1,2,..., \end{aligned}$$

where $\tau_2 = \tau_1 + 4M_1(2T+5)Ta$.

Therefore,

$$E\|x_t^n\|_{\mathcal{B}}^2 \le \tau_3 + 4M_1M_T(2T+5)b\int_0^t \sup_{0\le s\le T} E\left\|x^{n-1}(s)\right\|^2 ds, \quad n = 1,2,...,$$

where, $\tau_3 = \tau_2 + 4M_1T(2T+5)bN_T E\|\varphi\|_{\mathcal{B}}^2$.

since

$$E\left\|x_t^0\right\|_{\mathcal{B}}^2 \le M_1 E\|\varphi(0)\|^2 = \tau_4 \le \infty.$$

Thus,

$$E\|x_t^n\|_{\mathcal{B}}^2 \le \tau_5 \le \infty \text{ for all } n = 0,1,2,..., \text{ and } t \in [0,T]. \tag{5}$$

This proves the boundedness of $\{x^n(t), n \in \mathbb{N}\}$.

Let us next show that $\{x^n(t)\}$ is a cauchy sequence in $L_2(\Omega, \mathcal{B})$. For this, for $n, m \ge 1$, we have

$$\left\|x^{n+1}(t) - x^{m+1}(t)\right\|^2 \le 3M_1(T+5)\int_0^t \eta(\|x^n(s) - x^m(s)\|^2)ds.$$

Thus

$$\sup_{0\le s\le t} E\left\|x_s^{n+1} - x_s^{m+1}\right\|_{\mathcal{B}}^2 \le \tau_6\int_0^t \eta\left(\sup_{0\le r\le s} E\|x_r^n - x_r^m\|_{\mathcal{B}}^2\right)ds, \tag{6}$$

where $\tau_6 = 3M_1(T+5)$.

By integrating and applying Jensen's inequality in equation (6), we get

$$\begin{aligned} \int_0^t \sup_{0\le l\le s} E\left\|x_l^{n+1} - x_l^{m+1}\right\|_{\mathcal{B}}^2 ds &\le \tau_6\int_0^t\int_0^s \eta(\sup_{0\le r\le l} E\|x_r^n - x_r^m\|_{\mathcal{B}}^2)dlds \\ &\le \tau_6\int_0^t s\int_0^s \eta(\sup_{0\le r\le l} E\|x_r^n - x_r^m\|_{\mathcal{B}}^2)\frac{1}{s}dlds \\ &\le \tau_6 t\int_0^t \eta(\int_0^s \sup_{0\le r\le l} E\|x_r^n - x_r^m\|_{\mathcal{B}}^2\frac{1}{s}dl)ds. \end{aligned}$$

Then

$$\Phi_{n+1,m+1}(t) \quad \leq \quad \tau_6 \int_0^t \eta(\Phi_{n,m}(s))ds,$$

where

$$\Phi_{n,m}(t) \quad = \quad \frac{\int_0^t \sup\limits_{0\leq r\leq l} E\,\|x_r^n - x_r^m\|_{\mathcal{B}}^2\, ds}{t}.$$

From (5), it is easy to see that

$$\sup_{n,m} \Phi_{n,m}(t) \quad \leq \quad \infty.$$

so letting $\Phi(t) = \limsup_{n,m\to\infty} \Phi_{n,m}(t)$ and using the Fatou's lemma, we get

$$\Phi(t) \quad = \quad \tau_6 \int_0^t \eta(\Phi(s))ds.$$

Now, applying the Lemma 1, it tells that $\Phi(t) = 0$ for any $t \in [0,T]$. Thus, $\{x^n(t), n \in \mathbb{N}\}$ is a Cauchy sequence in $L_2(\Omega, \mathcal{B})$. so there is an $x \in L_2(\Omega, \mathcal{B})$ such that

$$\lim_{n\to\infty} \int_0^T \sup_{0\leq s\leq t} E\,\|x_s^n - x_s\|_{\mathcal{B}}^2\, dt \quad = \quad 0.$$

It is easy to follow that $E\,\|x_t\|_{\mathcal{B}}^2 \leq \tau_5$, by (5). Thus, $x(t)$ is a mild solution to (1). Further, By,(H2) and letting $n \to \infty$, we can claim that for $t \in [0,T]$

$$E\left\|\int_0^t R(t,s)\left[g(t,x_s^{n-1}) - g(t,x_s)\right] ds\right\|_{\mathcal{B}}^2 \quad \to \quad 0,$$

$$E\left\|\int_0^t R(t,s)\left[\sigma(t,x_s^{n-1}) - \sigma(t,x_s)\right] dw(s)\right\|_{\mathcal{B}}^2 \quad \to \quad 0,$$

$$E\left\|\int_0^t R(t,s)\left\|h(t,x_s^{n-1} - h(t,x_s))\right\| \tilde{N}(ds,du)\right\|_{\mathcal{B}}^2 \quad \to \quad 0.$$

Hence, taking limits on both sides of (4),

$$\begin{aligned} x(t) \quad &= \quad R(t,0)\varphi(0) + \int_0^t R(t,s)g(s,x_s^{n-1})ds + \int_0^t R(t,s)\sigma(s,x_s^{n-1})dw(s) \\ &+ \quad \int_0^t \int_Z h(s,x_s^{n-1},u)\tilde{N}(ds,du). \end{aligned}$$

Thus $x(t)$ is a mild solution of (1) on the interval $[0,T]$.

Now, we prove the uniqueness of the solutions of (1). Let $x, y \in L_2(\Omega, \mathcal{B})$ be the two solution of (1) on some interval $(-\infty, T]$. The uniqueness is obvious for $t \in (-\infty, 0]$. For $0 \leq t \leq T$, we have

$$E\,\|x_t - y_t\|_{\mathcal{B}}^2 \quad \leq \quad \tau_6 \int_0^t \eta(E\,\|x_s - y_s\|_{\mathcal{B}}^2)ds.$$

By using Bihari's inequality, we get

$$\sup_{t\in[0,T]} E\,\|x_t - y_t\|_{\mathcal{B}}^2 \quad = \quad 0, \quad 0 \leq t \leq T.$$

This complete the proof. □

The standard form of SFIDEs with poisson Jumps (2) on $t \in J = [0, T]$ is

$$x_\epsilon(t) = R(t,0)\varphi(0) + \epsilon \int_0^t R(t,s)g(s, x_{\epsilon_s})ds + \sqrt{\epsilon}\int_0^t R(t,s)\sigma(s, x_{\epsilon_s})dw(s) + \sqrt{\epsilon}\int_0^t R(t,s)h(s, x_{\epsilon_s}, u)\tilde{N}(ds, du) \quad (7)$$

Here the coefficients g, σ, h have same conditions as in (1) and $\epsilon \in [0, \epsilon_0]$ is a positive parameter with ϵ_0 a fixed number. According to the existence and uniqueness theorem of differential equations, for every fixed $\epsilon \in (0, \epsilon_0]$, equation (7) also has unique solution $x_\epsilon(t)$, $t \in [0, T]$. we impose some conditions on the coefficients to find whether the solution $x_\epsilon(t)$ will be approximated with small ϵ to some other simpler process.

Let $\hat{g}(x) : \mathcal{B} \to \mathbb{X}$, $\hat{\sigma}(x) : \mathcal{B} \to L_Q(\mathbb{Y}, \mathbb{X})$, and $\hat{h} : \mathcal{B} \to \mathbb{X}$ be borel measurable functions satisfying conditions (H2), (H3) and (H4). Further, we assume the following inequalities are satisfied:

(H5). For $x \in \mathcal{B}$ and $T_1 \in [0, T]$,

$$\begin{aligned} \frac{1}{T_1}\int_0^{T_1} \|g(s, x_s) - \hat{g}(x_s)\|^2 ds &\le \beta_1(T_1)(1 + \|x\|^2), \\ \frac{1}{T_1}\int_0^{T_1} \|\sigma(s, x_s) - \hat{\sigma}(x_s)\|^2 ds &\le \beta_2(T_1)(1 + \|x\|^2), \\ \frac{1}{T_1}\int_0^{T_1} \left\|h(s, x_s) - \hat{h}(x_s)\right\|^2 ds &\le \beta_3(T_1)(1 + \|x\|^2). \end{aligned}$$

where $\beta_i(T_1), i = 1,2,3$ are positive bounded functions with $\lim_{T_1 \to \infty} \beta_i(T_1) = 0$.

We now consider the following averaged SFIDEs with Poisson Jumps which correspond to the original standard form (7),

$$\begin{aligned} y_\epsilon(t) &= R(t,0)\varphi(0) + \epsilon\int_0^t R(t,s)\hat{g}(y_{\epsilon_s})ds + \sqrt{\epsilon}\int_0^t R(t,s)\hat{\sigma}(y_{\epsilon_s})dw(s) \\ &+ \sqrt{\epsilon}\int_0^t \int_Z R(t,s)\hat{h}(y_{\epsilon_s}, u)\tilde{N}(ds, du). \end{aligned} \quad (8)$$

Obviously, $y_\epsilon(t)$ and $x_\epsilon(t)$ have a unique solution on $[0, T]$ under the conditions (H2), (H3) and (H4). Now, we consider the relation between $x_\epsilon(t)$ and $y_\epsilon(t)$ in the following theorem.

Theorem 3.2. *Let the conditions $(H1) - (H5)$ hold. For a given arbitrary small number $\delta_1 \le 0$ and a constant $L \le 0$, $\alpha \in (0,1)$, there exists a number $\epsilon_1 \in (0, \epsilon_0]$ such that for all $\epsilon \in (0, \epsilon_1]$, we have*

$$E(\sup_{t \in [0, L\epsilon^{-\alpha}]} \|x_\epsilon(t) - y_\epsilon(t)\|^2) \le \delta_1. \quad (9)$$

Proof. We consider the difference of $x_\epsilon(t) - y_\epsilon(t)$. By (7) and (8), we have

$$\begin{aligned} x_\epsilon(t) - y_\epsilon(t) &= \epsilon\int_0^t R(t,s)\left[g(s, x_{\epsilon_s}) - \hat{g}(y_{\epsilon_s})\right] ds \\ &+ \sqrt{\epsilon}\int_0^t R(t,s)\left[\sigma(s, x_{\epsilon_s}) - \hat{\sigma}(y_{\epsilon_s})\right] dw(s) \\ &+ \sqrt{\epsilon}\int_0^t R(t,s)\left[h(s, x_{\epsilon_s}, u) - \hat{h}(y_{\epsilon_s}, u)\right]\tilde{N}(ds, du). \end{aligned}$$

Therefore, for $u \in [0, T)$, we obtain

$$\begin{aligned}
\sup_{0\le t\le u} \|x_\epsilon(t) - y_\epsilon(t)\|^2 &\le 3\epsilon^2 \sup_{0\le t\le u} \left\| \int_0^t R(t,s)\,[g(s,x_{\epsilon_s}) - \hat{g}(y_{\epsilon_s})]\,ds \right\|^2 \\
&+ 3\epsilon \sup_{0\le t\le u} \left\| \int_0^t R(t,s)\,[\sigma(s,x_{\epsilon_s}) - \hat{\sigma}(y_{\epsilon_s})]\,dw(s) \right\|^2 \\
&+ 3\epsilon \sup_{0\le t\le u} \left\| \int_0^t \int_Z R(t,s) \left[h(s,x_{\epsilon_s},u) - \hat{h}(y_{\epsilon_s},u)\right] \tilde{N}(ds,du) \right\|^2 . \\
&\le I_1 + I_2 + I_3.
\end{aligned}$$

Now,

$$\begin{aligned}
I_1 &= 3\epsilon^2 \sup_{0\le t\le u} \left\| \int_0^t R(t,s)\,[g(s,x_{\epsilon_s}) - g(s,y_{\epsilon_s}) + g(s,y_{\epsilon_s}) - \hat{g}(y_{\epsilon_s})]\,ds \right\|^2 \\
&\le 6\epsilon^2 \sup_{0\le t\le u} \left\| \int_0^t R(t,s)\,[g(s,x_{\epsilon_s}) - g(s,y_{\epsilon_s})]\,ds \right\|^2
\end{aligned}$$

$$+ \quad 6\epsilon^2 \sup_{0\le t\le u} \left\| \int_0^t R(t,s)\,[g(s,y_{\epsilon_s}) - \hat{g}(y_{\epsilon_s})]\,ds \right\|^2$$

$$\begin{aligned}
E(I_1) &\le 6\epsilon^2 E \sup_{0\le t\le u} tM_1 \int_0^t \|g(s,x_{\epsilon_s}) - g(s,y_{\epsilon_s})\|^2\,ds \\
&+ 6\epsilon^2 E \sup_{0\le t\le u} M_1 t^2 [\frac{1}{t} \int_0^t \|g(s,y_{\epsilon_s}) - \hat{g}(y_{\epsilon_s})\|^2\,ds] \\
&\le 6\epsilon^2 u M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds \\
&+ 6\epsilon^2 M_1 \sup_{0\le t\le u} \left\{ t^2 \beta_1(t)^2 [1 + E(\sup_{0\le s\le t} \|y_{\epsilon_s}\|^2)] \right\} \\
&\le 6\epsilon^2 u M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds \\
&+ 6\epsilon^2 M_1 u^2 \beta_1(u)^2 [1 + E(\sup_{0\le t\le u} \|y_{\epsilon_s}\|^2)].
\end{aligned}$$

By the properties of solutions, we know that $E\|x_0\|^2 < \infty$, then for each $t \le 0$, $E\|x(t)\|^2 < \infty$. This combines with the fact $\lim_{T_1\to\infty} \beta_1(T_1) = 0$.

Thus,

$$E(I_1) \le 6\epsilon^2 u M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds + 6\epsilon^2 M_1 u^2 \tau_7, \tag{10}$$

where $\tau_7 = \beta_1(u)^2[1 + E(\sup_{0\le t\le u} \|y_{\epsilon_s}\|)]$.

Similarly,

$$\begin{aligned}
E(I_2) &\le 6\epsilon M_1 E \left(\int_0^u \|\sigma(s,x_{\epsilon_s}) - \sigma(s,y_{\epsilon_s})\|^2\,ds \right) \\
&+ 6\epsilon M_1 E \left(\int_0^u \|\sigma(s,y_{\epsilon_s}) - \hat{\sigma}(y_{\epsilon_s})\|^2 \right) ds, \\
&\le 6\epsilon M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|)^2]ds + 6\epsilon M_1 u \beta_2(u)[1 + E(\sup_{0\le t\le u} \|y_{\epsilon_s}\|^2)].
\end{aligned}$$

By the properties of solutions, we know that $E\|x_0\|^2 < \infty$, then for each $t \leq 0$, $E\|x(t)\|^2 < \infty$. This combines with the fact that $\lim_{T_1\to\infty} \beta_2(T_1) = 0$.

Thus,

$$E(I_2) \leq 6\epsilon M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds + 6\epsilon M_1 u\tau_8, \tag{11}$$

where $\tau_8 = \beta_2(u)[1 + E(\sup_{0\leq t\leq t\leq u} \|y_{\epsilon_s}\|^2)]$.

Finally,

$$\begin{aligned}
I_3 &\leq 3\epsilon \sup_{0\leq t\leq u} \left\| \int_0^t \int_Z [h(s, x_{\epsilon_s}, u) - h(y_{\epsilon_s}, u)]\, \tilde{N}(ds, du) \right\|^2 \\
&= 6\epsilon \sup_{0\leq t\leq u} \left\| \int_0^t \int_Z R(t,s)\, [h(s, x_{\epsilon_s}, u) - h(s, y_{\epsilon_s}, u)] \right\|^2 v(du)ds \\
&+ 6\epsilon \sup_{0\leq t\leq u} \left\| \int_0^t \int_Z R(t,s) \left[h(s, y_{\epsilon_s}, u) - \hat{h}(y_{\epsilon_s}, u)\right] \right\|^2 v(du)ds \\
E(I_3) &\leq 6\epsilon M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds \\
&+ 6\epsilon M_1 u\beta_3(u)[1 + E(\sup_{0\leq t\leq u} \|y_{\epsilon_s}\|^2)].
\end{aligned}$$

By the properties of solutions, we know that $E\|x_0\|^2 \leq \infty$, then for each $t \leq 0$, $E\|x_t\|^2 \leq \infty$. This combines with the fact that $\lim_{T_1\to\infty} \beta_3(T_1) = 0$.

Thus

$$E(I_3) \leq 6\epsilon M_1 \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds + 6\epsilon M_1 u\tau_9, \tag{12}$$

where $\tau_9 = \beta_3(u)[1 + E(\sup_{0\leq t\leq u} \|y_{\epsilon_s}\|^2)]$.

Putting (10), (11) and (12) together, we see that

$$\begin{aligned}
\sup_{0\leq t\leq u} \|x_\epsilon(t) - y_\epsilon(t)\|^2 &\leq 6\epsilon M_1(\epsilon u + 2) \int_0^u E[\eta(\|x_{\epsilon_s} - y_{\epsilon_s}\|^2)]ds \\
&+ 6\epsilon M_1 u(\epsilon u\tau_7 + \tau_8 + \tau_9).
\end{aligned}$$

From the condition of concavity of η, we can find pair of constants a and b such that $\eta(x) \leq ax + b$, for all $x \leq 0$ and $\eta(0) = 0$. Substituting this, we get

$$\begin{aligned}
\sup_{0\leq t\leq u} \|x_{\epsilon_s}(t) - y_{\epsilon_s}(t)\|^2 &\leq 6a\epsilon M_1(\epsilon u + 2) \int_0^u E\|x_{\epsilon_s} - y_{\epsilon_s}\|^2 ds \\
&+ 6\epsilon u M_1[(\epsilon u + 2)b + (\epsilon u\tau_7 + \tau_8 + \tau_9)]
\end{aligned}$$

By Gronwall inequality,

$$\begin{aligned}
\sup_{0\leq t\leq u} \|x_\epsilon(t) - y_\epsilon(t)\|^2 &\leq 6\epsilon u M_1[(\epsilon u + 2)b + (\epsilon u\tau_7 + \tau_8 + \tau_9)] \\
&\exp \quad (6a\epsilon M_1(\epsilon u + 2)u).
\end{aligned}$$

Choose $\alpha \in (0,1)$ and $L > 0$ such that for every $t \in [0, L\epsilon^{1-\alpha}] \subset [0, T]$,

$$\sup_{0\leq t\leq u} \|x_\epsilon(t) - y_\epsilon(t)\|^2 \leq \tau_{10} L\epsilon^{1-\alpha}, \tag{13}$$

where $\tau_{10} = 6M_1[(L\epsilon^{1-\alpha} + 2)b + (L\epsilon^{1-\alpha}\tau_1 + \tau_2)]\exp(6aL\epsilon^{1-\alpha}M_1(L\epsilon^{1-\alpha}))$
is a constant.
That is, given any number $\delta_1 > 0$, we can choose $\epsilon_1 \in (0, \epsilon_0]$ such that for each $\epsilon \in (0, \epsilon_1]$ and for every $t \in [0, L\epsilon^{-\alpha}]$,

$$\sup_{0 \le t \le u} \|x_\epsilon(t) - y_\epsilon(t)\|^2 \quad \le \quad \delta_1.$$

This completes the proof.

Now, we give the theorem on convergence in probability. □

Theorem 3.3. *Let the conditions $(H1) - (H5)$ hold. For a given arbitrary small number $\delta_2 > 0$ and a constant $L > 0$, $\alpha \in (0, 1)$, there exists a number $\epsilon_1 \in (0, \epsilon_0]$ such that for all $\epsilon \in (0, \epsilon_1]$, we have*

$$\lim_{\epsilon \to 0} P(\sup_{t \in [0, L\epsilon^{-\alpha}]} \|x_\epsilon(t) - y_\epsilon(t)\| > \delta_2) = 0. \tag{14}$$

Proof. Using Chebyshev-Markov inequality and the result of the above theorem, for any given number δ_2, we have

$$\begin{aligned} \lim_{\epsilon \to 0} P(\sup_{t \in [0, L\epsilon^{-\alpha}]} \|x_\epsilon(t) - y_\epsilon(t)\| > \delta_2) \quad &\le \quad \frac{1}{\delta_2^2} E(\sup_{t \in [0, L\epsilon^{-\alpha}]} \|x_\epsilon(t) - y_\epsilon(t)\|^2) \\ &\le \quad \delta_2^2 \tau_{10} L\epsilon^{1-\alpha}. \end{aligned}$$

Taking limits on bothside of the inequality, we get the required result. Hence the proof. □

4 Stability

In this section, we study the stability through the continuous dependence on initial values.

Definition 4.1. *A mild solution $x(t)$ of the system (1) with initial value φ is said to be stable in the mean square if for all $\epsilon > 0$, there exists $\delta > 0$ such that*

$$E\,\|x_t - \hat{x}_t\|^2_{\mathcal{B}} \le \epsilon \text{ whenever } E\,\|\varphi - \hat{\varphi}\|^2 \le \delta \text{ for all } t \in [0, T]$$

where $\hat{x}(t)$ is another mild solution of the system (1) with initial data $\hat{\varphi}$.

Theorem 4.1. *Let the system (1) has φ_1 and φ_2 initial values. Let $x(t)$ and $y(t)$ be the mild solution of the system (1). If the hypothesis of theorem 1 is satisfied, then the mild solution of the system (1) is mean square stable.*

Proof. For $0 \le t \le T$, by the assumption for $x(t)$ and $y(t)$, we have

$$\begin{aligned} x(t) - y(t) \quad &= \quad R(t, 0)\,[\varphi_1(0) - \varphi_2(0)] \\ &+ \quad \int_0^t R(t, s)\,[f(s, x_s) - f(s, y_s)]\,ds \\ &+ \quad \int_0^t [R(t, s)g(s, x_s) - g(s, y_s)]\,dw(s) \\ &+ \quad \int_0^t \int_Z [h(s, x_s, u) - h(s, y_s, u)]\,\tilde{N}(ds, du) \end{aligned}$$

So, estmating as before, we get

$$E\|x_t - y_t\|^2_{\mathcal{B}} \le 4M_1E\|\varphi_1 - \varphi_2\|^2 + 4M_1(T+2)\int_0^t \eta(E\|x_s - y_s\|^2_{\mathcal{B}})ds.$$

Let $\eta_1(u) = 4M_1(T+2)\eta(u)$, where η is concave increasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that $\eta(0) = 0$, $\eta(u) > 0$ for $u > 0$ and $\int_{0^+}\frac{du}{\eta(u)} = +\infty$. So, $\eta_1(u)$ is obviously, a concave function from $\mathbb{R}^|$ to $\mathbb{R}^|$ such that $\eta_1(0) = 0$, $\eta_1(u) \ge \eta(u)$, for $0 \le u \le 1$ and $\int_{0^+}\frac{du}{\eta_1(u)} = +\infty$. Now for any $\epsilon > 0$, $\epsilon_1 = \frac{1}{2}\epsilon$, we have $\lim_{s\to 0}\int_s^{\epsilon_1}\frac{du}{\eta_1(u)} = \infty$. So, there is a positive constant $\delta < \epsilon_1$, such that $\int_\delta^{\epsilon_1}\frac{du}{\eta_1(u)} \le T$. Let

$$\begin{aligned} u_0 &= 4M_1E\|\varphi_1 - \varphi_2\|^2, \\ u(t) &= E\|x_t - y_t\|^2_{\mathcal{B}}, v(t) = 1, \end{aligned}$$

where $u_0 \le \delta \le \epsilon_1$. From Corollary 2.3, we have

$$\int_{u_0}^{\epsilon_1}\frac{du}{\eta_1(u)} \ge \int_\delta^{\epsilon_1}\frac{du}{\eta_1(u)} \ge T = \int_0^T v(s)ds.$$

So, for any $t \in [0,T]$, we estimate $u(t) \ge \epsilon_1$ holds. This completes the proof. □

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